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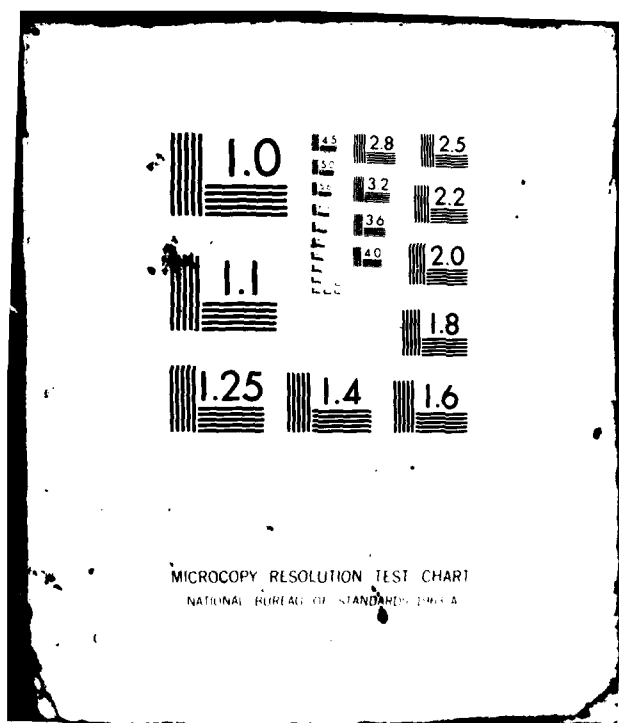
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Director
Office of Naval Research
Branch Office (N 2879)
666 Summer Street
Boston, MA 02210

Dear Sirs:

Enclosed please find Professor Fritz John's Final Technical Report for research conducted under ONR contract N00014-76-C-0301.

We trust this is satisfactory and urge you to contact me if anything further is required.

Sincerely,

Vito Rizzo

Vito Rizzo
Financial Administrator

VR/sg

cc: Prof. John
Ms. A. Greenberg
Mr. R. Alvarado
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| Asymptotic behavior of solutions, nonlinear equations and systems, higher order, wave equation, nonlinear wave propagation, blow-up of solutions, instability, hyperbolicity, systems of differential equations. | | |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) | | |
| F. John worked on two different topics during the period covered by the contract. The first one concerned the formation of singularities in nonlinear waves in several dimensions; the other one algebraic constraints implied by hyperbolicity of systems of partial differential equations with constant coefficients. ↗ | | |

RESEARCH IN CONTINUUM MECHANICS

CONTRACT NO. N-00014-76-C-0301

The final report consists of the progress reports of 1976 through 1980. In addition, Fritz John continued his work on the blow-up of solutions of hyperbolic equations. The main results follow.

F. John worked on two different topics during the period covered by the contract. The first one concerned the formation of singularities in nonlinear waves in several dimensions; the other one algebraic constraints implied by hyperbolicity of systems of partial differential equations with constant coefficients.

For the formation of singularities the number n of space dimensions is all important. For $n = 1$ solutions of genuinely nonlinear wave equations with initial data of compact support blow-up after a finite time T . Here T is of order $1/\epsilon$ for initial data of "amplitude" ϵ . F. J. showed (see [1]) that for $n \geq 3$ T is at least of order ϵ^{-2} . For equations of the type $u_{tt} - \Delta u = F(u)$ with $F(0) = F'(0) = 0$, $F''(0) > c > 0$ the lifespan T is exactly of order $1/\epsilon^2$ (see [2], [3]). S. Klainerman had shown that for equations of the type $u_{tt} - \Delta u = F(u', u'')$ no blow-up occurs for sufficiently small ϵ if $n > 5$. In [4] F. J. showed that Klainerman's theorem does not hold for $n = 3$; in the example $u_{tt} - \Delta u = 2u_t u_{tt}$ a solution with initial data of compact support and amplitude ϵ can blow-up for arbitrarily small ϵ . The equations considered play a role in nonlinear elasticity, and the

results indicate instability of certain elastic media.

For a single linear homogeneous partial differential equation with constant coefficients conditions for hyperbolicity take the form of inequalities satisfied by the coefficients. It is shown in [5], [6], [7] that for systems of equations hyperbolicity conditions can take the form of algebraic identities that must hold for the coefficients. F. J. studied in particular the case of three second order equations in four independent variables; it turns out that in significant cases the number of identities assuring hyperbolicity equals the number of double points of the characteristic surface. A consequence is that hyperbolic systems are very rare near some of the classical hyperbolic systems of mathematical physics.

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List of Publications

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- [1] Delayed singularity formation for solutions of nonlinear partial differential equations in higher dimensions, Proc. Nat. Acad. Sci. USA, Vol. 73, 2, pp. 281-282, February 1976.
- [2] Blow-up of solutions of nonlinear wave equations in three space dimensions, Proc. Natl. Acad. Sci. USA, Vol. 76, 4, pp. 1559-1560, April 1979.
- [3] Blow-up of solutions of nonlinear wave equations in three space dimensions, Manuscripta Mathematica 28, pp. 235-268, 1979.
- [4] Blow-up for quasi-linear wave equations in three space dimensions, Comm. Pure Appl. Math., Vol. 34, pp. 29-51, January 1981.
- [5] Restrictions on the coefficients of hyperbolic systems of partial differential equations, Proc. Natl. Acad. Sci. USA, Vol. 74, 10, pp. 4150-4151, October 1977.
- [6] Algebraic conditions for hyperbolicity of systems of partial differential equations, Comm. Pure Appl. Math., Vol. 31, pp. 89-106, March 1978.
- [7] Addendum to: Algebraic conditions for hyperbolicity of systems of partial differential equations, Comm. Pure Appl. Math., Vol. 31, pp. 787-793, November 1978.

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